MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

TMA1101 – CALCULUS

(All sections / Groups)

14 OCTOBER 2017
2:30 p.m. – 4:30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of six pages with **FIVE** questions.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.
4. <u>No calculators are allowed</u>.
5. **<u>You are required to write proper steps</u>.**

ANSWER ALL QUESTIONS.

QUESTION 1 [10 marks]

1 (a) Find the following limits.

[***You must show at least one intermediate step where*** $\lim\limits_{x\to c}$ ***is still needed.***]

(i) $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x^2-4}$

(ii) $\lim\limits_{x\to\infty}\dfrac{x^2+x-2}{3x^2+6x}$

[2 marks]

(b) Given $f(x)=\begin{cases} x+7, & \text{if } x<3 \\ 2x & \text{if } x=3 \\ x^2+1, & \text{if } x>3 \end{cases}$

(i) Find $f(3)$.

(ii) Determine $\lim\limits_{x\to 3^-} f(x)$ and $\lim\limits_{x\to 3^+} f(x)$.

[***For this part, you must show at least one intermediate step where*** $\lim\limits_{x\to 3^-}$ ***or*** $\lim\limits_{x\to 3^+}$ ***is still needed.***]

(iii) Does $\lim\limits_{x\to 3} f(x)$ exist? Give your reason. If it exists, state its value.

(iv) Is the function f continuous at 3? Give the reason for your answer.

[4.5 marks]

(c) (i) State the Intermediate Value Theorem (i.e., the full statement including the hypothesis and the conclusion).

(ii) Show that there is a root of the equation $x^4-3x^2-3=0$ in the interval $[1, 2]$.

You must write proper steps to arrive at the conclusion; just writing some calculations would not be enough.

[3.5 marks]

Continued ....

QUESTION 2 [10 marks]

(a) Use the formal definition of derivative to find $f'(3)$ when $f(x) = x^2 - x$.
You are reminded to write proper steps. [2.5 marks]

(b) Find $\dfrac{dy}{dx}$ with y as given.
[Use the product rule or the quotient rule for differentiation; show proper steps.]

(i) $y = \sqrt{x}\sin x$

(ii) $y = \dfrac{2x^3}{1+e^x}$

[3 marks]

(c) The point (4, 2) lies on the curve $3x + y^3 - xy = 12$.
Use implicit differentiation to obtain $\dfrac{dy}{dx}$ **in terms of x and y**.
Then determine the gradient of the tangent to the curve $3x + y^3 - xy = 12$ at the point (4, 2).

[4.5 marks]

Continued ....

QUESTION 3 [10 marks]

(a) (i) Use $\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$ to find the values of A and B which make the equation $\sin 4x\,\cos 5x = A\sin 9x + B\sin x$ an identity.

(ii) Evaluate $\int_0^{\pi} \sin 4x\,\cos 5x\,dx$

[3.5 marks]

(b) (i) Determine the values of A and B in the following partial fraction decomposition.

$$\frac{5x}{2x^2-3x-2} = \frac{A}{2x+1} + \frac{B}{x-2}$$

(ii) Integrate

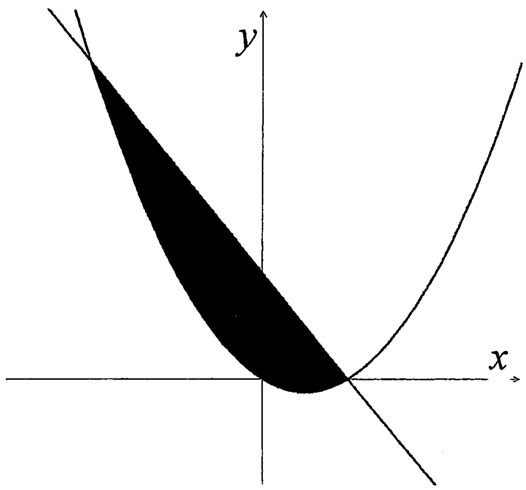
$$\int \frac{5x}{2x^2-3x-2}\,dx$$

[3 marks]

(c)

The figure shows a region bounded by the parabola $y = x^2 - x$ and the straight line $y = 2 - 2x$.

(i) Determine the x-coordinates of the points of intersection between the parabola and the straight line.



(ii) Write down a definite integral that can be used to find the area of this region and proceed to find the area.

[3.5 marks]

Continued ....

QUESTION 4 [10 marks]

(a) (i) Given an infinite series $\sum_{n=1}^{\infty} a_n$ with all terms positive and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$, discuss, according to the ratio test, how the value of L can be used to decide on the convergence or divergence of $\sum_{n=1}^{\infty} a_n$.

(ii) Let $a_n = \frac{3^n}{n^3}$. Determine $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

Then use the ratio test to determine whether the infinite series $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$ is convergent.

[3 marks]

(b) Find the Taylor polynomial of degree 3 for $f(x) = \ln x$ at $x = 1$.
Show proper steps.

[3 marks]

(c) A periodic function f with period 2π is defined as

$$f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ \pi, & 0 \le x < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le x < \pi \end{cases}$$

(i) Sketch the graph of f on the interval $[-2\pi, 2\pi]$.

(ii) The **Fourier series** of f has the form

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Find the value of b_3.

[4 marks]

Continued ....

QUESTION 5 [10 marks]

(a) Given $F(x, y) = x^2 + \cos(xy) - e^y$, find the partial derivatives $\dfrac{\partial F}{\partial x}$ and $\dfrac{\partial F}{\partial y}$.

[1 mark]

(b) Solve the first order **separable equation** $\dfrac{dy}{dx} = \dfrac{(x-1)(x+2)}{y(y+2)}$

subject to the initial condition $y(0) = 1$.

You may leave your answer in implicit form. [2.5 marks]

(c) You are told that e^x is an integrating factor for the first order linear equation

$\dfrac{dy}{dx} + y = 8$ subject to the initial condition $y(0) = 1$.

Solve the equation and give your solution in explicit form.

[3 marks]

(d) (i) Find the roots of the **characteristic equation** of the homogeneous differential equation

$$y'' - 3y' - 10y = 0$$

Then write down the general solution y_h of this homogeneous differential equation.

(ii) If $y = Ae^{3x}$ is a **particular solution** of the differential equation $y'' - 3y' - 10y = 2e^{3x}$. Determine the value of A.

(iii) Hence, write down the **general solution** for the differential equation $y'' - 3y' - 10y = 2e^{3x}$.

[3.5 marks]

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